Isolation of Stellar Signals in a Binary Occultation of Saturn's Rings (draft 4)

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Abstract

We describe a method of analyzing a binary star occultation from Cassini's Visual and Infrared Mapping Spectrometer (VIMS). This occultation is unique in that the binary star Alpha Centauri is occulted by Saturn's rings. We attempt to isolate the transmissions of Alp Cen A and B from the total signal, and use these data to search for longitudinal variations of Saturn's rings. To do this, we assume the signal is comprised of flux from Alp Cen A, Alp Cen B, reflected sunlight from the rings, and reflected Saturnshine from the rings. We also assume that each of these components has a radially dependent part and a wavelength dependent part. The first step we take is to normalize the data to correct for a spacecraft pointing variation. We solve for the wavelength components using gap edges, flux within gaps, and the opaque B ring. Once we know the wavelength dependent components, we can use four equations and Cramer's rule to solve for the radially dependent components. This allows us to examine the transmissions of Alp Cen A and Alp Cen B for longitudinal variations. Additionally, this analysis separates reflected Saturnshine and Sunlight from the signal, which has not been done before. The algorithm discussed in this paper could be applied to similar occultations with binary stars. It could also be used on single star occultations to extract the reflected Saturnshine and Sunlight components.

1 Description of the Occultation

1.1 Occultation Data

$$e^{\pi i} + 1 = 0 \tag{1}$$

The specific occultation discussed here was obtained with Cassini's Visual and Infrared Mapping Spectrometer (VIMS) on June 1, 2009. This is a flexible instrument that can operate in a number of different modes (Brown et al. 2004). The data discussed here were all obtained in occultation mode, where the imaging capabilities of the instrument are disabled, the short-wavelength VIS channel is turned off, and the IR channel obtains a continuous series of 0.8-5.1 μm spectra from a single pixel targeted at the star. To reduce data volume, the normal spectral resolution of the instrument is usually reduced by a factor of eight to 0.14 μm, with the spacecraft returning simultaneous time series for 31 spectral channels. (Bwakes paper)

When this occultation took place, the stellar positions projected onto the rings is approximately 32 km. Cassini's distance from the rings was approximately 780,000 km. The pixel size on VIMS is 0.25 x 0.50 mrad which translates to 200 x 400 km based off of Cassini's distance to Saturn. The sampling was done at approximately 20 milliseconds which corresponds to a radial resolution up to 100 meters.

1.2 Alpha Centauri

Alp Cen A is a type G2 V and Alp Cen B is a type K1 V. The spectra of the stars are only marginally different, but this difference is enough to separate the individual signals. This is discussed further in section 2.6. Alp Cen A is between 2 and 3 times brighter than Alp Cen B. Proxima Centauri is a type M5 V with a brightness of less than .01 percent of Alp Cen A. This star does not show up in our data. The separation of Alp Cen A and Alp Cen B is approximately 8.29 arcsec through a position angle of 237° , so both stars fall within the pixel. The diameter of Alp Cen A is about 9 milliarcsec which translates to a size of 32 m projected onto the rings. The Fresnel zone is $\sqrt{2\lambda D}$ which is between 40 and 90 m for this occultation.

2 Separating Transmissions of Alp Cen A and B

2.1 The Raw Equation and Normalization

(fitstar.pro, normalize.pro) For this occultation, we assume that the total raw flux ($I'(\lambda, r)$) at a given wavelength and radius is composed of four parts. Flux from Alp Cen A ($I'_a(\lambda, r)$), flux from Alp Cen B

($I'_b(\lambda, r)$), reflected sunlight from the rings ($I'_V(\lambda, r)$), and reflected Saturnshine from the rings ($I'_U(\lambda, r)$).

$$I'(\lambda, r) = I'_a(\lambda, r) + I'_b(\lambda, r) + I'_V(\lambda, r) + I'_U(\lambda, r) \tag{2}$$

At radii outside (141,000-146,000 km or r_A) and inside (70,000-74,000 km or r_D) of Saturn's rings, we expect $I'(\lambda, r_A) = I'(\lambda, r_D)$ because we should see only starlight. In reality, the values of $I'(\lambda, r_A)$ and $I'(\lambda, r_D)$ differ by an average of 1.59 percent for the first 10 channels. This implies that the stellar baseline is not stable, probably due to a spacecraft pointing variation during the occultation. However, we can correct this through normalization. We assume a linear drift through time. This line provides an estimate of what the lightcurve would look like if there were no rings blocking the stars, or the unocculted data. At each wavelength, we fit a line to the median flux at r_A and r_D to obtain:

$$I_a(max) + I_b(max) = m(\lambda)R(r) + b(\lambda) \tag{3}$$

We then divide the observed signal by this linear fit. This corrects for the drift in the combined stellar flux.

$$I(\lambda, r) = \frac{I'(\lambda, r)}{m(\lambda)R(r) + b(\lambda)} = I_a(\lambda, r) + I_b(\lambda, r) + I_V(\lambda, r) + I_U(\lambda, r) \tag{4}$$

These normalized data now has the desired property that the signal inside and outside of Saturn's rings are equal. Furthermore, the maximum value of the stellar signal is now 1 at all radii. Now that the stellar baseline is constant, we assume that each component to be the product of one radially dependent and one wavelength dependent part. The total signal from Alp Cen is the sum of the transmission of Alp Cen A ($T_a(r)$) multiplied by a wavelength dependent factor ($A(\lambda)$) and the transmission of Alp Cen B ($T_b(r)$) multiplied by a wavelength dependent factor ($B(\lambda)$). The total light reflected from Saturn's rings is the signal of reflected sunlight ($V(r)$) multiplied by a wavelength dependent factor ($\alpha(\lambda)$), and the ring signal of reflected Saturnshine ($U(r)$) multiplied by a wavelength dependent factor ($\beta(\lambda)$).

$$I(\lambda, r) = A(\lambda)T_a(r) + B(\lambda)T_b(r) + \alpha(\lambda)V(r) + \beta(\lambda)U(r) \tag{5}$$

After this normalization, we know that outside the rings $V = U = 0$ and $T_a = T_b = 1$ so the equation turns into $A(\lambda) + B(\lambda) = I(\lambda, r_A) = I(\lambda, r_D) = 1$, which is true for all wavelengths. The two transmissions, T_a and T_b, vary quite rapidly because they are only limited by the temporal resolution. This temporal resolution allows for details on the order of 1 km. On the other hand, U and V vary smoothly because their resolution is based off of the spatial resolution which is approximately 200x400 km. A and B are the star's spectra as viewed from VIMS. The term α is the product of the solar spectrum and the ring's spectrum divided by Alpha Centauri's spectrum. Finally, β is the product of Saturn's spectrum and the ring's spectrum divided by Alpha Centauri's spectrum.

Assuming equation **??** is an accurate description of the flux, we can try to find values for the wavelength dependent components, A, B, α, and β. Once we know all of the wavelength dependent components, we can choose four different wavelengths to create four independent equations and solve for the four radially dependent factors. Isolating T_a, T_b, V, and U will allow us to search for differences between T_a and T_b. These differences would indicate longitudinal variations within the rings. Additionally, we can how Sunlight and Saturnshine changes throughout the rings.

2.2 Obtaining constants A and B

(getabvalues.pro, viewabdetails.pro)

In an occultation, we see sharp drops in signal corresponding to when a star passes behind an opaque edge in the rings. However, in the Alpha Centauri occultation, we see two distinct drops in the signal for each ring edge. From these edges, we can gather information about A and B. There are three distinct areas with radii r_1, r_2, and r_3 corresponding to levels 1,2, and 3 respectively (see fig **??**).

In step 1, both stars are shining without hinderance, therefore, $T_a(r_1) = T_b(r_1) = 1$. At 2, I assume that $T_a(r_2) = 1$ while $T_b(r_2) = T \neq 1$. Finally, in r_3, if the signal is not wildly varying, we also assume that $T_a(r_3) = T_b(r_3) = T_b(r_2) = T$. Because of the large pixel size, the ring signal varies smoothly so that between r_1 and r_3. U and V are not dependent on radius, therefore, $V(r_1) = V(r_2) = V(r_3) = V$ and $U(r_1) = U(r_2) = U(r_3) = U$. Dropping the λ to make the equations simpler, the equations now read as follows:

$$I(r_1) = A + B + \alpha V + \beta U \tag{6}$$

$$I(r_2) = A + BT + \alpha V + \beta U \tag{7}$$

$$I(r_3) = T(A + B) + \alpha V + \beta U \tag{8}$$

$$I(r_1) - I(r_2) = B(1 - T) \tag{9}$$

$$I(r_2) - I(r_3) = A(1 - T) \tag{10}$$

$$I(r_1) - I(r_3) = (A + B)(1 - T) = (1 - T) \tag{11}$$

Recall that, $A+B$ is equal to 1 after normalization. We divide Eqns. **??** and **??** by **??** to get A and B individually. This method will also work when there is a dense ringlet within a gap, such as the Huygens ringlet. We used the edge of the A ring, inner keeler gap, inner encke gap, outer Huygen's ringlet, and inner Huygen's ringlet. A table of the radii for the three levels at each gap is in table ???.

For some of the edges, the transmission is varying slightly over r_2 and r_3. If this variation is large compared to the difference in flux between r_2 and r_3, then the edge was thrown out. See table ??? for the full list.

We can only separate the signals if we can differentiate between the two stars from their spectra. If they have the same spectra, we would not be able to separate them from the same signal. To see if the two spectra are different, we plot A/B in Figure **??**. Additionally, we plotted what A/B would look like based off of the stars known temperatures and radii. This line is very close to the result that we got with the analysis described above.

2.3 Obtaining α

(getallalphas.pro viewallalphas.pro) This occultation was taken from the dark side of Saturn's rings, but Sunlight reflects off of the rings and contributes a significant amount of flux in these data. Similar to the stellar components, we have assumed that the reflected Sunlight is composed of a wavelength dependent component, $\alpha(\lambda)$, and a radially dependent component, $V(r)$. The α parameter is the ratio of sunlight reflected from the rings to the combined stellar signals. Because of the physical restrictions, we cannot isolate U. We can only solve for Uα.

Within gaps, the total normalized flux is significantly higher than 1, as seen in Figure **??**. This indicates a large amount of light from the rings. These regions are gaps with radius r_g. We assume that all of this excess is from light reflected from Saturn's rings with an insignificant contribution from Saturnshine. So, within the gaps we have $T_a(r_g) = T_b(r_g) = 1$, $U(r_g) = 0$ and $V(r_g) \neq 0$. We chose outside the A ring, Keeler Gap, Encke Gap, Jeffreys Gap, Russel Gap, Herschel Gap, and the Maxwell Gap. For the following equations, the term r_g indicates a small range of radii; The value associated with that range, $I(\lambda, r_g)$, is the median over the radial ranges listed in Table 2.

$$I(\lambda, r_g) = \alpha(\lambda)V(r_g) + A(\lambda) + B(\lambda) = \alpha(\lambda)V(r_g) + 1 \tag{12}$$

$$\frac{I(\lambda_x, r_g) - 1}{I(\lambda_1, r_g) - 1} = \frac{\alpha(\lambda_x)V(r_g)}{\alpha(\lambda_1)V(r_g)} = \frac{\alpha(\lambda_x)}{\alpha(\lambda_1)} \tag{13}$$

From this method, we are able to calculate ratios of α for all possible wavelength combinations. If α is constant with radius, then each gap should give us values of α similar to all other gaps. However, we find that in different areas in the ring system, the values of α are systematically shifted for different areas of the ring. This indicates that α is not quite independent of radius. In figure **??**, we plot $\frac{\alpha(\lambda_x)}{\alpha(\lambda_1)}$ Vs. wavelength for several areas of the ring. The α values in the C Ring and Cassini Division are consistantly higher than α values in the A ring. This is probably due to different chemistry throughout the rings.

We choose to simply average the α values for the A ring and Cassini Division together. In doing this, we create a systematic error based on radius. This error will show up as an overall drift in the values of the other components. Since we are only interested in the small scale variations in T_a and T_b, we can continue, knowing that the value of the two transmissions will not be accurate, but the small scale differences between them should be sensible.

2.4 Obtaining β

(getallbetas.pro) Since this occultation is from the dark side of the rings, both sunlight and starlight are completely blocked in several areas in the very opaque B ring. For certain radii (r_b) , we assume that the only signal contribution comes from βU, and $T_a(r_b) = T_b(r_b) = U(r_b) = 0$.

Out of the 4 parameters, β is the most straightforward to obtain. In dividing median filtered signals from two different channels, we obtain ratios of β. Although reflected Saturnshine is a very small factor in Eq. (**??**), it was nearly trivial to find ratios of the β parameter.

$$I(\lambda, r_b) = \beta(\lambda)U(r_b) \tag{14}$$

$$\frac{I(\lambda_x, r_b)}{I(\lambda_1, r_b)} = \frac{\beta(\lambda_x)U(r_b)}{\beta(\lambda_1)U(r_b)} = \frac{\beta(\lambda_x)}{\beta(\lambda_1)} \tag{15}$$

2.5 Mathematically Separating the Signal

(gettransmission.pro) To separate Eq. 1 into it's four radially dependent components, we choose four wavelengths and create four linearly independent equations to solve for $Ta(r), Tb(r), U(r)$, and $V(r)$. For four wavelengths,$\lambda_1, \lambda_2, \lambda_3$, and λ_4, we rewrite EQ1 in matrix form as follows:

$$\begin{pmatrix} I(\lambda_1, r) \\ I(\lambda_2, r) \\ I(\lambda_3, r) \\ I(\lambda_4, r) \end{pmatrix} = \begin{pmatrix} A(\lambda_1) & B(\lambda_1) & \alpha(\lambda_1) & \beta(\lambda_1) \\ A(\lambda_2) & B(\lambda_2) & \alpha(\lambda_2) & \beta(\lambda_2) \\ A(\lambda_3) & B(\lambda_3) & \alpha(\lambda_3) & \beta(\lambda_3) \\ A(\lambda_4) & B(\lambda_4) & \alpha(\lambda_4) & \beta(\lambda_4) \end{pmatrix} \begin{pmatrix} T_a(r) \\ T_b(r) \\ V(r) \\ U(r) \end{pmatrix} \tag{16}$$

Recall that we cannot solve for any α or β individually, but we can know ratios of α or β respectively. Because of this, it was necessarry to divide the 3^{rd} column by α_1 and the fourth column by β_1. To keep the equations equal, the α_1 and β_1 are multiplied with V and U in the bottom two equations. We now take Eq. (**??**) and solve for $T_a, T_b, \alpha_1 V$, and $\beta_1 U$ using Cramer's rule and some simple matrix column operators.

$$T_a(r) = \frac{\begin{vmatrix} I(\lambda_1, r) & B(\lambda_1) & 1 & 1 \\ I(\lambda_2, r) & B(\lambda_2) & \frac{\alpha(\lambda_2)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_2)}{\beta(\lambda_1)} \\ I(\lambda_3, r) & B(\lambda_3) & \frac{\alpha(\lambda_3)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_3)}{\beta(\lambda_1)} \\ I(\lambda_4, r) & B(\lambda_4) & \frac{\alpha(\lambda_4)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_4)}{\beta(\lambda_1)} \end{vmatrix}}{\begin{vmatrix} A(\lambda_1) & B(\lambda_1) & 1 & 1 \\ A(\lambda_2) & B(\lambda_2) & \frac{\alpha(\lambda_2)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_2)}{\beta(\lambda_1)} \\ A(\lambda_3) & B(\lambda_3) & \frac{\alpha(\lambda_3)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_3)}{\beta(\lambda_1)} \\ A(\lambda_4) & B(\lambda_4) & \frac{\alpha(\lambda_4)}{\alpha(\lambda_1)} & \frac{\beta(\lambda_4)}{\beta(\lambda_1)} \end{vmatrix}} \tag{17}$$

$$T_a = -\frac{\begin{matrix} -\beta_1\alpha_2 I_3 B_4 + \beta_1\alpha_2 B_3 I_4 - \beta_1 B_3\alpha_4 I_2 - \beta_1\alpha_3 B_2 I_4 + \beta_1 I_3\alpha_4 B_2 + \beta_1\alpha_3 I_2 B_4 \\ -\alpha_2\beta_3 B_1 I_4 + \alpha_2 I_3\beta_4 B_1 + \alpha_2\beta_3 I_1 B_4 - \alpha_2 B_3\beta_4 I_1 + \beta_3\alpha_1 B_2 I_4 + \alpha_3 B_2\beta_4 I_1 \\ -I_3\beta_2\alpha_4 B_1 - I_3\alpha_1\beta_4 B_2 - \alpha_3 I_2\beta_4 B_1 - \alpha_3\beta_2 I_1 B_4 + \alpha_3\beta_2 B_1 I_4 - \beta_3\alpha_1 I_2 B_4 \\ +\alpha_1\beta_2 I_3 B_4 - \beta_3 I_1\alpha_4 B_2 - B_3\beta_2\alpha_1 I_4 + B_3\beta_2\alpha_4 I_1 + B_3\alpha_1\beta_4 I_2 + \beta_3 B_1\alpha_4 I_2 \end{matrix}}{\begin{matrix} \beta_2 A_4\alpha_1 B_3 - \beta_2 A_1\alpha_4 B_3 + \beta_2\alpha_4 A_3 B_1 - \beta_2\alpha_3 A_4 B_1 - \beta_2\alpha_1 A_3 B_4 - A_1\alpha_2\beta_3 B_4 \\ +\beta_3\alpha_1 A_2 B_4 + \alpha_1 A_3\beta_4 B_2 - \beta_4 A_2\alpha_1 B_3 - \beta_3\alpha_4 A_2 B_1 - A_4\alpha_2\beta_1 B_3 + \beta_1 A_2\alpha_4 B_3 \\ +\alpha_2\beta_1 A_3 B_4 + A_1\alpha_2\beta_4 B_3 - \alpha_4 A_3\beta_1 B_2 + \beta_2\alpha_3 A_1 B_4 + \alpha_3\beta_4 A_2 B_1 + \alpha_3 A_4\beta_1 B_2 \\ -\alpha_3\beta_1 A_2 B_4 - \alpha_3 A_1\beta_4 B_2 - A_4\beta_3\alpha_1 B_2 - \alpha_2\beta_4 A_3 B_1 + \alpha_2\beta_3 A_4 B_1 + A_1\beta_3\alpha_4 B_2 \end{matrix}} \tag{18}$$

To find the best 4 wavelengths to use, we maximized the determinant of the denominator in Equation **??**. If A/B is close to a constant, this determinant would be zero, which would make the equation unsolvable.

We have finally separated the signal into four radially dependent components. In the original data, there were three distinct levels (see Figure 1) at ring gaps such as the Encke and Keeler gaps, but in the profile of T_a we see one drop, and in T_b, we see a similar drop, but shifted. Because we used 4 channels to do this analysis, the random noise in T_a and T_b is multiplied by about 4 from $I(r, \lambda)$. We reduce the noise further by using the fact that αV and βU should be smooth functions. We use a boxcar smoothing function and smooth the αV and βU by 200 points, or about 20 km. Next, we subtracted the smoothed αV and βU from each signal. This allows us to set up two equations to solve for T_a and T_b.

fin!

Gap Name	Level 1 (km)	Level 2 (km)	Level 3 (km)
A Ring Edge	136,769.75 - 136,766.06	136,765.14 - 136,762.38	136,760.94 - 136,758.79
Keeler Gap Outer*	136,519.93 - 136,517.17	136,523.62 - 136,520.85	136,526.08 - 136,524.23
Keeler Gap Inner	136,485.43 - 136,482.66	136,482.05 - 136,478.98	136,478.37 - 136,476.52
Encke Gap Right*	133,742.54 - 133,737.66	133,745.60 - 133,743.15	133,748.04 - 133,747.13
Encke Gap Left*	133,427.83 - 133,424.16	133,423.55 - 133,420.81	133,420.50 - 133,418.97
Huygen's ringlet Outer	117,852.37 - 117,851.20	117,850.62 - 117,850.33	117,849.74 - 117,848.28
Huygen's ringlet Inner	117,834.07 - 117,832.31	117,835.23 - 117,834.65	117,836.99 - 117,835.53
88,700 Gap	88,705.970 - 88,701.075	88,712.127 - 88,706.444	88,714.258 - 88,712.364
87,500 Gap*	87,492.741 - 87,490.181	87,499.184 - 87,493.672	87,502.210 - 87,499.649

Table 1: This table lists the radii for all the levels considered in equation . A median is used over these ranges to obtain the flux used in Equations ??, ??, and ??. The * indicates and edge that was not averaged into the final value for A and B. This is due to the flux difference between level 1 and level 3 being too small.

Gap Name	Inner Radius	Outer Radius
Outside of A ring 1	136,856.00	136,900.00
Outside of A ring 2	136,812.00	136,856.00
Outside of A ring 3	136,768.00	136,812.00
Keeler Gap	136,487.00	136,511.00
Encke Gap 1	133,600.00	133,700.00
Encke Gap 2	133,434.00	133,477.00
Jeffreys Gap	118,931.00	118,963.00
Russell Gap	118,601.00	118,625.00
Herschel Gap 1	118,268.00	118,284.00
Herschel Gap 2	118,194.00	118,229.00
Maxwell Gap 1	87,584.80	87,605.40
Maxwell Gap 2	87,393.30	87,468.70

Table 2: alpha caption goes here too!!!

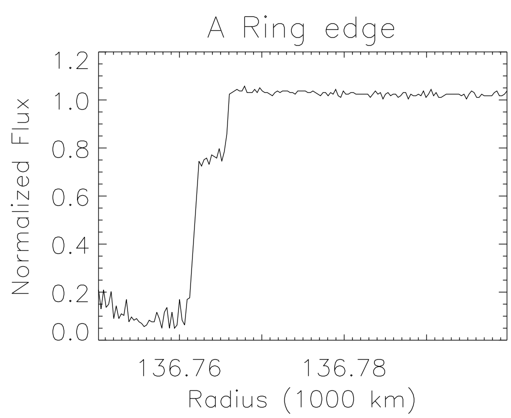

Figure 1: This shows the 3 levels in the outer edge of the a ring. Note how the signal is relatively constant for each step, indicating a constant transmission. We did not include gaps where the flux difference between r_1 and r_3 was less than 0.7 which left 5 gaps.

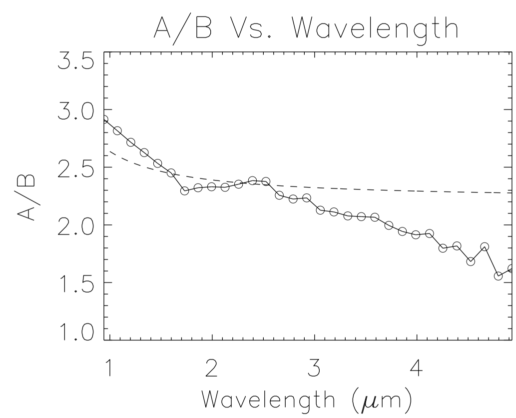

Figure 2: This shows A/B Vs. Wavelength after averaging the A and B values of the 5 gaps together. The dashed line shows what A/B would look like based only on temperature and radius. Since the dashed line is close to the

Channel number	Percent	Channel number	Percent
0	1.51268	16	3.1437
1	1.39412	17	3.48595
2	1.36328	18	3.74157
3	1.68789	19	4.26571
4	2.16118	20	4.48457
5	2.40631	21	5.14176
6	1.22578	22	4.87828
7	1.15858	23	4.98275
8	1.15049	24	6.3262
9	1.64562	25	7.36996
10	1.81376	26	4.87255
11	2.24811	27	18.2993
12	2.98956	28	23.6303
13	2.88676	29	50.0253
14	3.02731	30	49.0934
15	3.26258		

Table 3: This is the percent difference between raw flux outside and inside the rings. If there was no spacecraft pointing variation, then this drift would be close to 0. However, we see a drift in all wavelengths. We correct this drift through our normalization process.

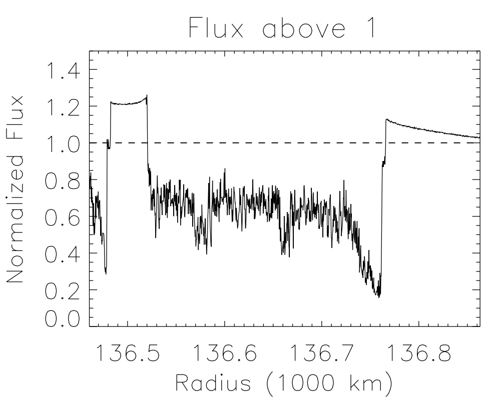

Figure 3: After normalizing, the total stellar flux does not exceed 1, but in some places in the ring we see some areas that have signal of more than 1. Here we see the keeler gap and the edge of the A ring. The ring signal outside of the rings smoothly increases. The dashed line indicates where a normalized flux of 1 is.

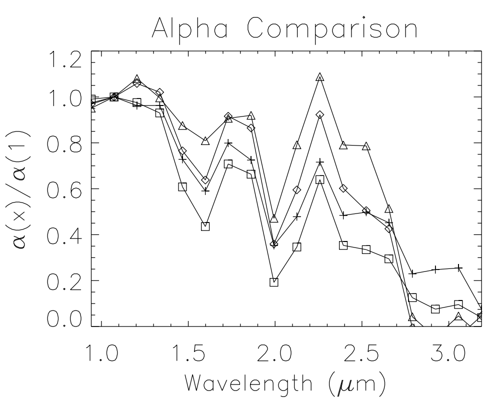

Figure 4: This shows the comparison of alpha values obtained for different parts of the ring.

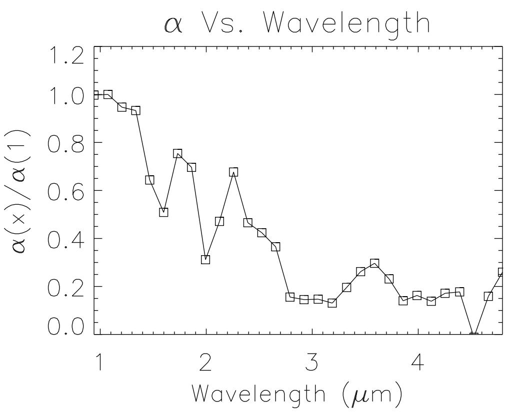

Figure 5: This shows us the final accepted value of alpha after averaging the 4 lines in the other graph. Based on previous observations, we expected ice absorption bands at 1.55, 2.0, and 3.0 microns.

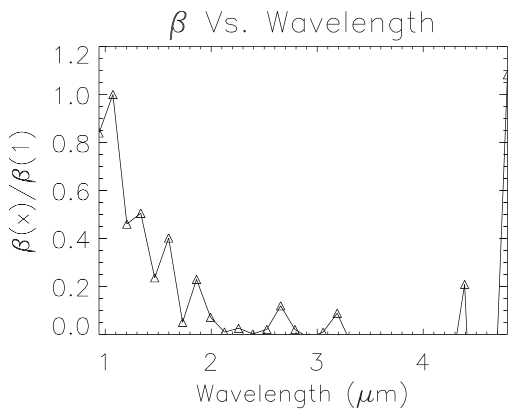

Figure 6: This shows us the final accepted value of beta. We expected to see methane absorption bands at 1.4, 1.7, and 2.3, combined with the ring absorption bands at 1.55, 2.0, and 3.0 microns.

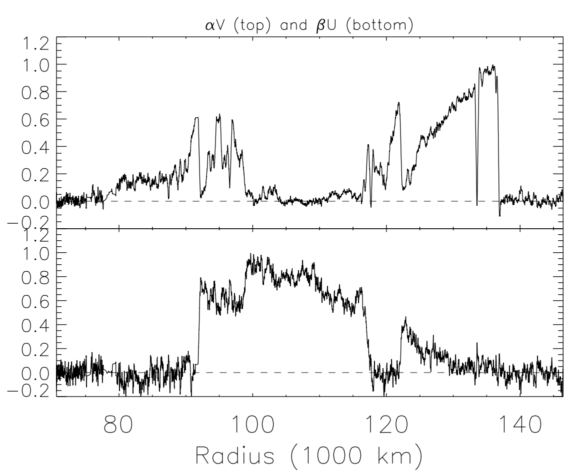

Figure 7: This figure shows both V and U divided by their own respective maximums.

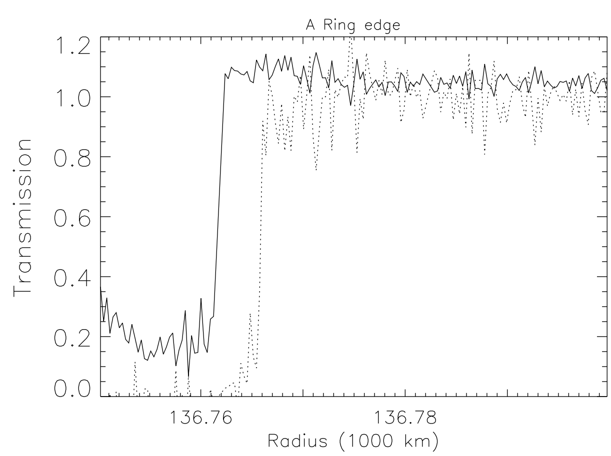

Figure 8: This shows the how the 2 steps in the previous figure are now separated into individual levels.